Finding Discontinuities Algebraically Transcript

We’ve looked at some of the most common types of functions
and discussed where each will be continuous.
Now let's look at some examples of how we can use this function
knowledge to come up with points at which
some specific functions are discontinuous without having
to first sketch the graph.
As our first example, we want to find
all values of x where the function is discontinuous for f of x equals the quantity 4x minus 3
over the quantity 2x minus 7.
First, we need to decide which family of functions
f belongs to.
Since we see a fraction consisting of a numerator
and denominator made up of polynomials,
we can conclude that f is a rational function.
Now let's remind ourselves about continuity
for rational functions.
A rational function y equals p of x
over q of x will be continuous for all x
where the denominator q of x is unequal to 0.
To find points of discontinuity, then, we
need to determine when the denominator will equal 0.
For this example, f of x will be discontinuous when 2x minus 7
equals 0.
If we add 7 to both sides, we get 2x equals 7.
Dividing both sides by 2 gives x equals 7/2.
We can conclude that f of x is discontinuous
when x equals 7/2.
The next example asks us to find all values
of x squared g of x equals e to the power 2x minus 3
is discontinuous.
Looking at g of x, we can see that it's
an exponential function with a base of e.
Recall that when an exponential function is
of the form y equals a to the x, it's continuous for all x.
In turn, we can say that g of x is
continuous for all real powers of e.
Since 2x minus 3 will always be a real number when
x is a real number, we can say that g
of x is continuous for all real values of x.
This means there are no values where f of x is discontinuous.
The third example is to find all values
of x where f of x equals the square root of the quantity
3x minus 1 is discontinuous.
Function f is the square root function.
As a reminder, when we have a square root function
with a radicand of ax plus b, the function
will be continuous for all x where
ax plus b is greater than or equal to 0,
since we can't take an even root of a negative number.
A given function will then be discontinuous whenever
ax plus b is less than 0.
For this example, we can say that f of x
is discontinuous whenever 3x minus 1 is less than 0.
Adding 1 to both sides gives 3x is less than 1.

Dividing both sides by 3 gives that x is less than 1/3.
We can conclude that f of x is discontinuous for all values of x such that x is less than 1/3.
The solution to this last example included what we call an open interval, since it doesn't include the value 1/3 as part of the solution.
An interval that includes both of its endpoints is called a closed interval.
We can make some general statements about continuity on open and closed intervals.
We say a function f is continuous on an open interval a, b, if it is continuous at every x value in the interval.
Notice that we don't need continuity at either the a or the b, since they're not included in the interval.
Closed intervals are a little different, since they include the end points a and b in the interval.
We say a function f is continuous on a closed interval from a to b if one, it's continuous on the open interval from a to b; two, it's continuous from the right at x equals a; and three, it's continuous from the left at x equals b.
But wait a minute.
What does it mean to be continuous from the right or from the left?
Continuous from the right at x equals a means that as we approach a from the right, we'll not encounter any discontinuities.
And continuous from the left at x equals b means that as we approach b from the left, we won't encounter any discontinuities.
Let's look at an example to illustrate the idea of continuity on a closed interval.
Is the function f of x equals the square root of 1 minus x squared continuous on the closed interval from negative 1 to 1?
The graph of this function is the upper half of the circle of radius 1 centered at the origin.
We have closed circles at the end points, negative 1, 0, and 1, 0, to indicate that they are included in the interval.
We'll check the three conditions for continuity on a closed interval.
The first condition that the function be continuous in the open interval is met, because we don't need to lift a pencil in order to sketch the curve.
And so we can say f of x is continuous on the open interval from negative 1 to 1.
Now, we need to check each of the endpoints.
When we approach negative 1 from the right, we see that we get the limit as x approaches negative 1 from the right of f of x is equal to 0, which is equal to f of negative 1.
f is continuous from the right at x equals negative 1.
Approaching x equals 1 from the left gives us the limit as x approaches 1

from the left of f of x is equal to 0, which is equal to f of 1.
f is continuous from the left at x equals 1.
We've satisfied all three conditions of continuity
on a closed interval.
So we can conclude that f of x is continuous on the closed
interval from negative 1 to 1.
Verifying continuity on any closed interval
means we just have to remember the extra step of checking
an appropriate one-sided limit for each endpoint.